Linear Algebra II

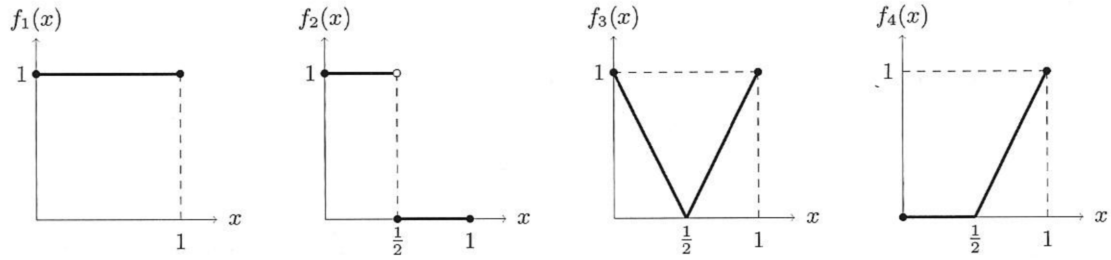
19/03/2015, Thursday, 14:00-16:00

1 $(3+8+3+8+8=30$ pts) **Inner product spaces**

Consider the vector space $C[0,1]$ with the inner product

$$\langle f,g\rangle = \int_0^1 f(x)g(x)\,dx.$$

Let f_1, f_2, f_3 and f_4 be given by as follows:



(a) Is the set $\{f_1, f_2, f_3\}$ an orthonormal set?

(b) Compute $\langle f_i, f_j\rangle$ where $i,j \in \{1,2,3,4\}$. (HINT: You may want to use the relationship between the integral of and the area under a curve.)

(c) Find the angle between f_1 and f_2.

(d) Apply Gram-Schmidt process to obtain an orthonormal basis for the subspace spanned by f_1, f_2 and f_3. (HINT: You may want to plot each function the process computes.)

(e) Find the closest function to f_4 in the subspace spanned by f_1, f_2 and f_3.

2 $(10+10=20$ pts) **Eigenvalues**

(a) Let A be a square matrix and p be a polynomial. Show that if x is an eigenvector of A corresponding the eigenvalue of λ then x is also an eigenvector of $p(A)$. Find the corresponding eigenvalue.

(b) Let A and B be nonsingular matrices of the same size. Show that AB and BA have the same eigenvalues.

3 $(4+8+8=20$ pts) **Diagonalization**

(a) Let A be a nonsingular matrix. Show that if A is diagonalizable then so is A^{-1}.

(b) Let

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

where a, b, c, and d are real numbers. Determine all values of (a, b, c, d) such that M is unitarily diagonalizable.

(c) Let

$$A = \begin{bmatrix} p & q \\ -q & p \end{bmatrix}$$

where p and q are real numbers. Find a unitary diagonalizer for A.

4 $(15+5=20$ pts) **Singular value decomposition**

(a) Compute a singular value decomposition of the matrix

$$M = \begin{bmatrix} 2 & -4 \\ 2 & 2 \\ -4 & 0 \\ 1 & 4 \end{bmatrix}.$$

(b) Find the closest (with respect to Frobenius norm) matrix of rank 1 to M.

10 pts free